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INTRODUCTION

The purpose of this paper is to present a model for stiffening rings under tension which may be applied to the shells (housings) of hermetically sealed compressors. While the characteristics of zero tension stiffening rings that are welded to the shell are basically well understood [1], the investigation of rings under pretension requires a theory that can predict the natural frequencies and modes of such a ring as function of its tension. This is a required input if the receptance approach [1] is to be used to estimate stiffening effectiveness. When the authors worked in the unrelated application field of the vibration of automotive tires, they found that commonly used ring models applied to tire under tension [2,3] (the inflation pressure causes a tension in the load carrying belt) had the flaw that they would not predict lower mode frequencies well. Particularly, for the rigid body mode of $u_3 = A\cos\theta$, the n = 1 mode, the ring model would predict a non-zero influence of tension, which clearly cannot be. Translated to a compressor shell, for a mode of the form $\Phi(x)\cos\theta$, a ring stiffener should have zero influence. But currently used ring models do not conform to this requirement.

To make a long story short, the present paper presents briefly an extended ring model which will satisfy all requirements. Results of this ring model can then be used in the receptance method of reference [1] to predict, at least for circular, cylindrical housings, what the influence of tension might be. In this paper, this influence is discussed qualitatively, reviewing in the process the rule of stiffening as described in reference [1].

A reasonable question is, of course, if tension can be effectively introduced in a ring in a cost effective and reliable way. It is not at all clear if this is possible. Also, the practical results observed during consulting activities of the senior author were inconclusive. Uncontrolled experiments (lets try it and see if it works) based on steel bands wrapped under tension around cylindrical housings would sometimes work and sometimes not. What makes the issue even more complicated is that it was not clear if the ring bands under tension were slipping relative to the compressor housing or not. Slipping is, of course, a damping mechanism and a measured sound pressure level reduction may be due to it and not to the de-tuning effect a stiffener normally provides.

Therefore, this paper has to be viewed to be only a small stepping stone toward a full investigation of this issue.

SHIFTING NATURAL FREQUECIES BY STIFFENING

Here, the basic concept of stiffeners, as discussed in reference [1], is reviewed for the example of a closed cylindrical shell. The natural frequencies of a cylindrical shell with a ring stiffeners is given by

$$\alpha_{11} + \beta_{11} = 0 \tag{1}$$

where α_{11} is the line receptance of the shell at the line of stiffener attachment and β_{11} is the line receptance of the stiffening ring. The line receptance of an equivalent, simply supported, cylindrical shell representing the closed compressor shell can be shown to be

$$\alpha_{11} = \frac{2}{\rho_s h_s L} \sum_{m=1}^{\infty} \frac{1}{\omega_{mn}^2 - \omega^2} \sin^2 \frac{m\pi x^*}{L} \tag{2}$$

where L = height of shell, ρ_s = mass density, h_s = thickness of shell, ω_{mn} are the natural frequencies of the shell before stiffening is applied, n = 0, 1, 2, ··· is the mode component number in circumferential direction, m = 0, 1, 2, ··· is the mode component number in axial direction, ω are the as yet unknown new natural frequencies of the stiffened shell and x^* is the location in axial direction where the stiffening ring is applied.

The stiffening ring receptance β_{11} is [1]

$$\beta_{11} = \frac{1}{\rho_s A\left(\omega_n^2 - \omega^2\right)} \tag{3}$$

where ρ_s = mass density of the ring material, A = cross-section area, ω_n are the natural frequencies of the ring by itself before it is attached to the shell and the ω are the as yet unknown natural frequencies of the stiffened shell. Therefore, the equation

$$\frac{2}{\rho h L} \sum_{m=1}^{\infty} \frac{1}{\omega_{mn}^2 - \omega^2} \sin^2\left(\frac{m\pi x^*}{L}\right) + \frac{1}{\rho_s A\left(\omega_n^2 - \omega^2\right)} = 0 \tag{4}$$

has to be searched for the values of ω which satisfy it.

To gain an intuitive insight, one can investigate the case where there is only a small stiffening effect such that ω is just a perturbation of the original ω_{mn}. This allows one to write approximately, because only the one term in the series where ω is closest to ω_{mn} dominates,

$$\omega^2 = \omega_{mn}^2 \frac{1 + \left(\frac{2M_s}{M}\right)\left(\frac{\omega_n}{\omega_{mn}}\right)^2 \sin^2\left(m\pi \frac{x^*}{L}\right)}{1 + \left(\frac{2M_s}{M}\right)\sin^2\left(m\pi \frac{x^*}{L}\right)} \tag{5}$$

where M_s and M are the total masses of the ring stiffener and the shell, respectively:

$$M_s = 2\pi a A \rho_s \tag{6}$$

$$M = 2\pi L a h \rho \tag{7}$$

This approximated solution shows immediately that the new shell natural frequency ω_k is increased or decreased according to the following law:

$$\begin{cases} \omega_k > \omega_{mn} & if \quad \omega_n > \omega_{mn} \\ \omega_k < \omega_{mn} & if \quad \omega_n < \omega_{mn} \end{cases} \tag{8}$$

as pointed out in reference [1] where a partial shell was used as example. This means that the natural frequencies of the stiffening ring by itself have to be higher than the natural frequencies of the unstiffened shell to have any beneficial effect. Based on the consulting experience of the senior author, this basic condition of stiffening is often violated in practice (and to the surprise of the designers, the stiffening ring has no beneficial effect).

Thus, the goal is to increase the natural frequencies of the stiffening ring. Among other measure, one way is to tension the ring. If this is feasible from an economic viewpoint is not clear, because the ring would either have to be shrunk on the shell or it would have to be tightened by some kinematic arrangement before it is welded to the shell (or perhaps left unwelded). This has to be determined in the future.

NATURAL FREQUENCIES OF STIFFENING RINGS UNDER TENSION

In the following, a ring theory is outlined which meets the requirement of the case investigated here. It was derived by the authors when developing vibration models of automotive tires [2], which can be viewed as rings under tension on elastic foundations. It was noted by the authors that existing ring models described the tension effect incompletely, causing an inconsistency and errors for lower modes. This work is briefly reviewed in the following and the improved formula for the natural frequencies of a ring under tension is given.

Basically, the improved approach required an improved description of "hoop" strain in the ring, $\varepsilon^{o}_{\theta\theta}$:

$$\varepsilon^{o}_{\theta\theta} = \frac{1}{a}\left(u_3 + \frac{\partial u_\theta}{\partial \theta}\right) + \frac{1}{2a^2}\left(u_3 + \frac{\partial u_\theta}{\partial \theta}\right)^2 + \frac{1}{2a^2}\left(\frac{\partial u_3}{\partial \theta} - u_\theta\right)^2 \tag{9}$$

where a = mean radius of ring, u_3 = transverse deflection, and u_θ = circumferential deflection. Utilizing Hamilton's principle, the following equations were derived [2] (here, the elastic foundation and rotation, important for tire model, are eliminated):

$$-\frac{\partial N_{\theta\theta}}{\partial \theta} - \frac{N^r_{\theta\theta}}{a}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{N^r_{\theta\theta}}{a}u_\theta - 2\frac{N^r_{\theta\theta}}{a}\frac{\partial u_3}{\partial \theta} - \frac{1}{a}\frac{\partial M_{\theta\theta}}{\partial \theta} + \rho h a\,\ddot{u}_\theta = a q_\theta \tag{11}$$

$$N_{\theta\theta} + N^r_{\theta\theta} + \frac{N^r_{\theta\theta}}{a}u_3 - \frac{N^r_{\theta\theta}}{a}\frac{\partial^2 u_3}{\partial \theta^2} + 2\frac{N^r_{\theta\theta}}{a}\frac{\partial u_\theta}{\partial \theta} - \frac{1}{a}\frac{\partial^2 M_{\theta\theta}}{\partial \theta^2} + \rho h a\,\ddot{u}_3 = a q_3 \tag{12}$$

where $N_{\theta\theta}$ and $M_{\theta\theta}$ are the force and moment resultants [2] and $N^r_{\theta\theta}$ is the tension resultant per unit width. Evoking the well tested assumption that the elastic extension of the reference surface of the ring (the "neutral" plane) is very small, yields the equation of motion in terms of transverse displacement only:

$$\begin{aligned}&\frac{D}{a^4}\left(\frac{\partial^6 u_3}{\partial \theta^6} + 2\frac{\partial^4 u_3}{\partial \theta^4} + \frac{\partial^2 u_3}{\partial \theta^2}\right) - \frac{N^r_{\theta\theta}}{a^2}\frac{\partial^4 u_3}{\partial \theta^4} - 2\frac{N^r_{\theta\theta}}{a^2}\frac{\partial^2 u_3}{\partial \theta^2} - \frac{N^r_{\theta\theta}}{a^2}u_3 \\ &+ \rho h\left(\frac{\partial^4 u_3}{\partial t^2 \partial \theta^2} - \frac{\partial^2 u_3}{\partial t^2}\right) = \frac{\partial q_\theta}{\partial \theta} + \frac{\partial^2 q_3}{\partial \theta^2}\end{aligned} \tag{12}$$

where D = bending stiffness. To obtain the natural frequencies, we substitute (setting $q_\theta = q_3 = 0$)

$$u_3(\theta, t) = A_n e^{j(n\theta + \omega t)} \tag{13}$$

Setting $D = Eh^3/12$ and $I = bh^3/12$, where h = ring thickness, b = ring width, E = Young's modulus, I = moment of inertia, we obtain after some manipulation:

$$\omega_n = \frac{\left(n^2-1\right)}{a\sqrt{\rho hb\left(n^2+1\right)}}\sqrt{\frac{n^2EI}{a^2}+T} \tag{14}$$

where $T = bN_{\theta\theta}^{r}$ is the tension force. It should be noted that for the n = 1 mode, the natural frequency is zero, which is as it should be because it corresponds to a motion $u_3 = A\cos\theta e^{j\omega t}$, which is a rigid body mode for the ring. It is a well known fact that ring stiffeners do not work for n = 1 modes, tension or no tension. Thus, the formula is entirely consistent. This is an improvement over some previous ring theories [3].

RESULTS AND DISCUSSION

Let us take as example a stiffening ring where a=100mm, $\rho=7.85\times10^{-9}$Nsec/mm^4, b=30mm, h=2mm or 4mm, E=241MPa, and plot ω_n as function of n and T (note that $T \leq \sigma_{Y.P.}hb$). This is shown in Figure 1. The various lines in Figure 1 show, down below, the natural frequencies for the first four natural frequencies of the ring, as they increase with increasing tension. We see that tensioning the stiffening ring is more effective for thin rings then thick rings because in the later the EI term dominates.

In Figure 2, an example of a circular cylindrical housing shell is shown, of dimensions h_s = 2mm, a = 100mm, L = 200mm, the material being steel. The thickness of the stiffening ring, without tension (T=0) is not enough to raise the natural frequencies of the m=1 modes of the shell, as a matter of fact the original natural frequencies are in general lowered (because the thin ring violates the stiffening law that $\omega_k > \omega_{mn}$ [1]). Only when the tension is increased to 15,000 and 30,000N does stiffening become effective in the bending dominated region to the right of n=4. Natural frequencies to the left of n=4 are actually lowered, but they tend to be less easily excited (for a discussion of peculiarities of curved structures such as shell, which behave fundamentally differently than beams and plates, see reference [1]). While the shift in natural frequencies may appear small to some observers, it should be remembered that resonances tend to come in very sharp and a small detuning is often sufficient to cause a noticeable improvement of noise. In Figure 3, it is shown that, for T=14,000N and h_s=2mm, the higher m-branch frequencies are actually lowered. One cannot expect that a given stiffening ring that starts to stiffen certain m=1 branch frequencies will also act as a stiffener for the higher m-branch frequencies. Figure 4 illustrates that using thicker stiffening rings is a more effective way of raising natural frequencies in the bending dominated stiffening region. It also illustrates that even for the thicker ring, the lowest natural frequencies value is hardly changed. Therefore, the n-value of an objectionable mode should be known before attempting stiffening. It also illustrates that thicker stiffening rings are in general more desirable, under tension or not.

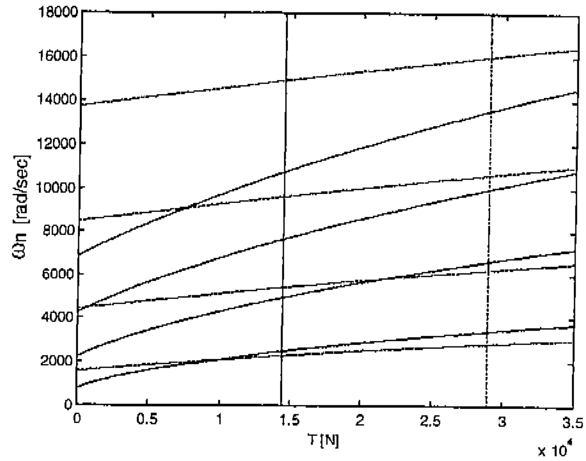

Figure 1 Natural frequencies of stiffening ring: — h = 2mm, -- h = 4mm

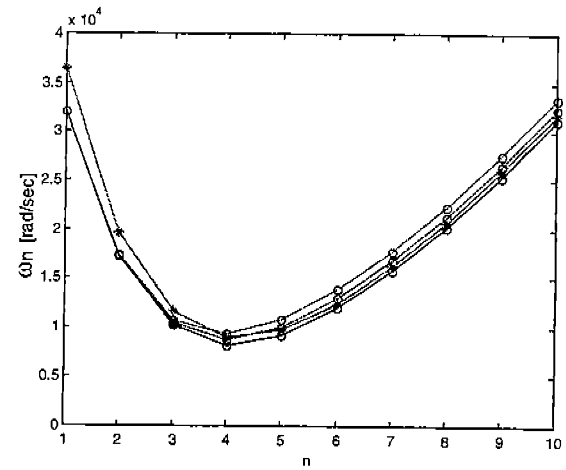

Figure 2 Natural frequencies of the cylindrical shell for hs=2mm, m=1:
(—:T=0N, ---:T=15000N, ⋯:T=30000N, o:stiffened, *:unstiffened)

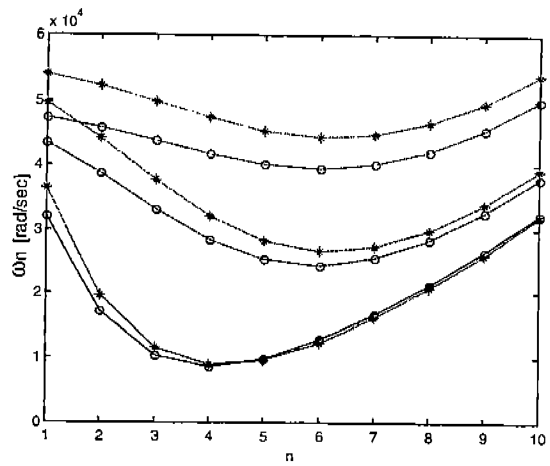

Figure 3 Natural frequencies of the cylindrical shell for T=14000N and hs=2mm :
(—:m=1, ---:m=3, ⋯:m=5, o:stiffened, *:unstiffened)

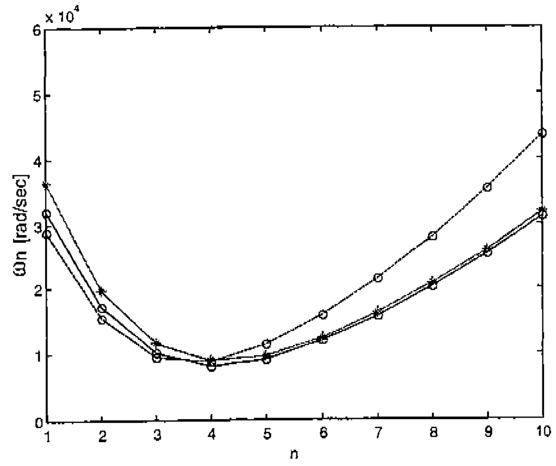

Figure 4 Natural frequencies of the cylindrical shell for T=14000, m=1:
(—:hs=2mm, ---:hs=4mm, o:stiffened, *:unstiffened)

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